TRINITY COLLEGE

Year 12 Mathematics Applications
Test 2 2019

Section 1 Calculator Free

Sequences

STUDENT'S NAME: Solutions

DATE: Friday 29th March **TIME:** 20 minutes **MARKS**: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

State the recursive rule for this geometric sequence: 380, 190, 95, 47.5…

$\frac{190}{380} = \frac{95}{190} = 0.5$

$T_{n+1} = 0.5T_n, \quad T_1 = 380$

✓ recursive
✓ $r = 0.5$
✓ $T_1 = 380$

2. (3 marks)

Determine the first three terms of the sequence:

$$T_{n+1} = T_n + 2(n+1), \quad T_1 = 6$$

$T_1 = 6$, $T_2 = 6 + 2(1+1) = 10$ $T_3 = 10 + 2(3) = 16$

✓ T_1
✓ T_2
✓ T_3

3. (7 marks)

A sequence is such that $T_1=c$, $T_2=31$, $T_3=59$ and $T_4=115$.

(a) Show that the sequence is neither an AP or a GP. [2]

$\frac{115}{59} \neq \frac{59}{31}$ ✓ $\therefore$ not GP

$115-59 \neq 59-31$ ✓ $\therefore$ not AP

must show sums

The sequence can be defined using the linear recurrence model, $T_{n+1} = aT_n + b$, $T_1=c$.

(b) Determine the values of the constants a and b. [3]

① $115 = 59a + b$ ✓

② $59 = 31a + b$

$59 - 31a = 115 - 59a$

$59a - 31a = 115 - 59$

$28a = 56$

$a = 2$ ✓

$b = 59 - 31(2)$

$= 59 - 62$

$= -3$

$\therefore a = 2$

$b = -3$

✓ pair of eqns
✓ solves eqns
✓ subs to find b

(c) Hence, or otherwise, state the recursive rule for this sequence. [2]

$T_{n+1} = 2T_n - 3$

$T_2 = 2T_1 - 3$

$31 = 2T_1 - 3$

$34 = 2T_1$

$T_1 = 17$

$\therefore c = 17$

So $T_{n+1} = 2T_n - 3$, $T_1 = 17$

✓ uses eqn from (b)
✓ solves for c

4. (8 marks)

The fourth term and ninth term of an arithmetic sequence are 81 and 41 respectively.

(a) Determine the first term of this sequence. [3]

$d = \frac{41-81}{9-4}$

$= \frac{-40}{5}$

$= -8$

$T_1 = 81 - 3(-8)$

$= 81 + 24$

$= 105$

✓ calculates d

✓ uses d value

✓ $T_1 = 105$

(b) State the rule for the n^{th} term of the sequence. [2]

$T_n = 105 - 8(n-1)$

✓ a value

✓ d value

(c) How many positive terms are there in this sequence? [3]

$0 > 105 - 8n + 8$

$0 > 113 - 8n$

$\frac{-113}{-8} < n$

$n > 14\frac{1}{4}$ so 15^{th} term is negative

$\therefore$ 14 terms

(or =)

✓ general eqn < 0

✓ solves eqn

✓ interprets solution

TRINITY COLLEGE

Year 12 Mathematics Applications
Test 2 2019

Section 2 Calculator Assumed

Sequences

STUDENT'S NAME: Solutions

DATE: Friday 29th March **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (8 marks)

(a) The first three terms, in order, of a geometric sequence are 44, 17.6 and 7.04.

(i) Deduce a rule for the n^{th} term of this sequence. [3]

$\frac{17.6}{44} = r$ $\quad r = \frac{2}{5}$ $\quad T_n = 44 \times \left(\frac{2}{5}\right)^{n-1}$ or $T_n = 44 \times 0.4^{n-1}$

✓ determines r
✓ general rule
✓ correct general rule

(ii) Calculate the 5th term of this sequence. [1]

$T_5 = 1.1264$ or $\frac{704}{625}$ ✓

(b) The first three terms, in order, of an arithmetic sequence are 89, 81 and 73.

(i) State a recursive rule for this sequence. [3]

$81 - 89 = -8$

$T_{n+1} = T_n - 8, \; T_1 = 89$

✓ calculates d
✓ recursive rule
✓ states T_1

(ii) State the value of the first non-positive term. [1]

$T_{13} = -7$

✓ states -7

6. (7 marks)

The population of a rare species of blue and green Trin frog has been declining over recent years. In 2014 an initiative was put in place to try and stabilise the population of frogs. It is known that the population decreases by 60% each year and so 150 frogs will be added to combat the population decrease. The initial population of frogs in 2014 was 2000.

(a) Show that the population of frogs one year after 2014, ie. in 2015, was 950 frogs. [2]

$2000 \times 0.4 + 150 = 950$

✓ uses $r = 0.4$
✓ shows calculation

(b) State a recursive rule that models the number of frogs n years after 2014. [2]

$T_{n+1} = 0.4T_n + 150$, $T_0 = 2000$

✓ correct recursive rule
✓ $T_0 = 2000$

(c) Using your rule from (b) state the predicted population of frogs in 2020. [2]

$T_6 = 257.168$

$\therefore$ 257 frogs

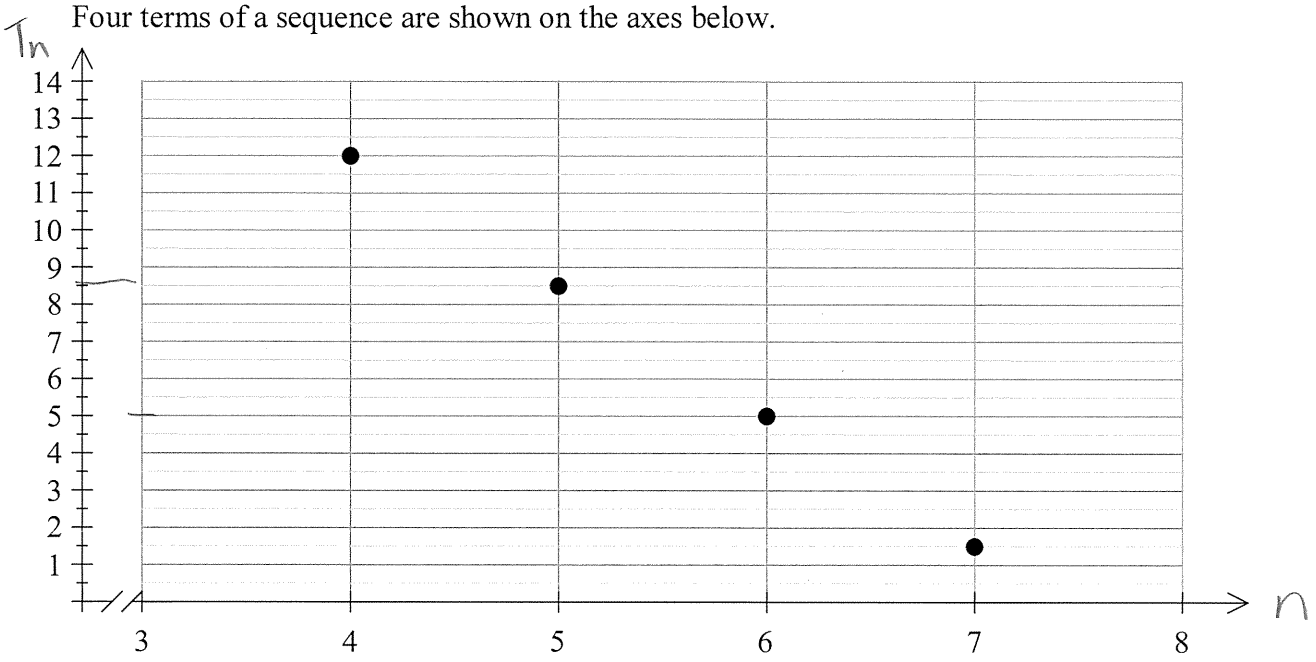
✓ states T_6
✓ rounds answer

(d) Determine the long-term frog population. [1]

250 frogs ✓

7. (7 marks)

Four terms of a sequence are shown on the axes below.



(a) Clearly label the axes using T_n and n. [1]

✓ labels both axes

(b) Show that the sequence forms an arithmetic progression. [1]

$T_4 = 12$, $T_5 = 8.5$, $T_6 = 5$

$5 - 8.5 = 8.5 - 12$ ✓

$-3.5 = -3.5$

(c) State a simplified general rule for T_n. [3]

$d = -3.5$

$a = 12 + 3(3.5)$

$= 22.5$

$T_n = 22.5 - 3.5(n-1)$

$= 26 - 3.5n$

✓ calculates a
✓ states general rule
✓ simplifies

(d) State the value of n such $T_n = -100$ [2]

$-100 = 26 - 3.5n$

$n = 36$

✓ states eqn to solve
✓ solves eqn

note: can be solved on classpad

8. (8 marks)

Lachlan is planning a trip to Italy at the end of the year. His grandparents agree to give him some spending money. They plan to set up an account and make weekly deposits. They offer him the choice of two different savings plans.

Choice A : \$2.00 in the first week, \$2.20 in the second week, \$2.42 in the third week and so on.

Choice B : \$20 per week

(a) Write a recursive rule for A_n, the amount Lachlan receives, in terms of A_{n-1}, if he picks choice A. [3]

$r = \frac{2.20}{2} = 1.1$ $A_n = 1.1A_{n-1}, A_1 = 2$

✓ calculates r
✓ writes rule in terms of A_n & A_{n-1}
✓ states A_1

(b) In which week are the values of the deposit of Choice A and of Choice B the closest? [1]

$A_{25} = 19.70$ $B_{25} = 20$
$A_{26} = 21.70$ $B_{26} = 20$

∴ week 25 ✓

(c) In which week does the total value of the savings for Choice A become greater than that of Choice B? [1]

Sum of A = 802.50
Sum of B = 780

∴ week 39

Lachlan's grandparent's have decided that they will make weekly deposits for 45 weeks.

(d) State, with reasoning, which savings plan Lachlan should opt for. [3]

The total for A would be 1437.81 compared to Choice B which would be 900, so Lachlan should opt for choice A.

✓ calculates value of A after 45 weeks
✓ calculates value of B after 45 weeks
✓ interprets results